

LORENTZ VIOLATION IN THE LINEARIZED GRAVITY

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We study some consequences of the introduction of a Lorentz-violating modification term in the linearized gravity, which leads to modified dispersion relations for gravitational waves in the vacuum. We also discuss possible mechanisms for the induction of such a term in the Lagrangian.

1. Introduction

In recent years, a great amount of work has been done exploring the possibility of very small departures of Lorentz invariance, the Standard-Model Extension proposed by Kostelecký and Colladay¹ furnishing a general background for such investigations. One of the interesting possibilities arising in such a context is the modification of the dispersion relations governing the propagation of particles in the vacuum, which could generate outstanding effects in astrophysical observations, for instance.² For gravity theories, the incorporation of Lorentz violation is delicate, in particular when the violation is not spontaneous;³ nevertheless, it is worthwhile to investigate how to introduce Lorentz violation in gravity in such a way that simple physical effects can be studied. This is one of the objectives of our work.

We consider the linearized gravity theory augmented by one Lorentz violating term in the Lagrangian and show that it leads to a modified dispersion relation for the propagation of gravitational waves in the vacuum. Next, we discuss one possible mechanism for the generation of such a term, based on a deformation of the Poisson algebra of the metric fluctuation and its canonical conjugated momenta.⁴ More details of this calculation,

together with another method to generate the particular form of Lorentz violation we consider here, are discussed elsewhere.⁵

2. The modified Fierz-Pauli action

The starting point of our study is the inclusion of a Lorentz-violating term ΔL ,

$$\Delta L = -2\epsilon^{\lambda\mu\nu\rho}\theta_\rho h_{\nu\sigma}\partial_\lambda h_\mu^\sigma, \quad (1)$$

in the Einstein-Hilbert Lagrangian in the weak field approximation, also known as the Fierz-Pauli Lagrangian. Here, $h_{\mu\nu}$ is the metric fluctuation, and $\theta^\rho = (0, \theta^i)$, $i = 1, 2, 3$ is a parameter for the Lorentz violation. The term ΔL breaks the gauge invariance of the model (which is the linearized version of the original diffeomorphism invariance of General Relativity). Since there exists a preferred direction in spacetime, the angular momentum is no longer conserved, however, the energy-momentum tensor is still conserved since the background (Minkowski) spacetime is homogeneous. As for the Bianchi identities, they are not satisfied, and this problem can be traced back to the general incompatibility between explicit Lorentz violation and Riemann-Cartan geometry.³

To study gravitational wave propagation, we show that the equations of motion for the traceless transversal part of the metric fluctuation $\tilde{h}_{\mu\nu}$ are gauge invariant and decouple from the other components of $h_{\mu\nu}$; this happens if $\theta_{ik}\partial_k\tilde{h}_{ij} = 0$. If this condition holds, we end up with

$$\frac{1}{2}\square\tilde{h}_{ij} + 2\left[\theta_{ik}\dot{\tilde{h}}_{kj} + \theta_{jk}\dot{\tilde{h}}_{ki}\right] = 0. \quad (2)$$

Here, we defined an antisymmetric symbol $\theta_{ij} = -\epsilon_{0ijk}\theta^k$.

The above-mentioned condition can be met if we choose $\theta^i = (0, 0, \theta/4)$, such that the only nonvanishing θ_{ij} are $\theta_{12} = -\theta_{21} = -\theta/4$, and consider a wave propagating in the x_3 direction. In this case, Eq. (2) can be solved by the ansatz $\tilde{h}_{ij} = H_{ij}e^{iq^\mu x_\mu}$, $q = (E, \vec{p})$, and we end up with only two independent equations, $\square Z \pm 2i\theta\dot{Z} = 0$, where $Z = H_{11} - iH_{12}$, $\bar{Z} = H_{11} + iH_{12}$. The corresponding dispersion relations are given respectively by $E = -\theta \pm \sqrt{p^2 + \theta^2}$ and $E = +\theta \pm \sqrt{p^2 + \theta^2}$, with $p = |\vec{p}|$. Thus the dispersion relations are modified, exhibiting a kind of birefringence phenomenon for the two circular polarizations of the gravitational waves.

3. The generation of ΔL

One possibility for the induction of Eq. (1) is based on the deformation of the Poisson algebra of the canonical variables.^{4,6} The inspection of the

constraint structure of the Fierz-Pauli action yields a set of primary constraints $\Phi_\mu^{(1)} = p_{0\mu} \simeq 0$, and secondary ones $\Phi_j^{(2)} = \partial_i p_{ij} \simeq 0$ and $\Phi_0^{(2)} = \partial_l \partial_l h_{kk} - \partial_i \partial_j h_{ij} \simeq 0$, which act as generators of the gauge symmetry.

The proposed deformation consists of assuming the noncommutativity of the canonical conjugated momenta, $\{p_{ij}(\vec{x}), p_{kl}(\vec{y})\} = \theta_{ijkl} \delta(\vec{x} - \vec{y})$, where θ_{ijkl} is a symbol possessing symmetry $\theta_{1234} = \theta_{2134} = \theta_{1243} = -\theta_{3412}$. We then require that, within this modified algebra, the secondary constraints still generates the same gauge symmetries of the undeformed theory. To this end, we have to modify the secondary constraints as follows,

$$R_k = \partial_i p_{ik} - \theta_{klnm} \partial_l h_{nm}. \quad (3)$$

This modification implies a modification of the Hamiltonian of the model, and therefore of the Lagrangian, which turns out to be

$$L_{\text{new}} = L_{\text{FP}} - \left[2\theta_{jlnm} \partial_l h_{nm} + \frac{1}{2} \dot{h}_{ij} \theta_{klkj} h_{kl} \right]. \quad (4)$$

To verify whether this procedure can reproduce ΔL , we make an assumption on the form of the θ_{jlnm} . The most appropriate choice seems to be

$$\theta_{ijkl} = \theta_{ik} \tilde{\delta}_{jl} + \theta_{il} \tilde{\delta}_{jk} + \theta_{jl} \tilde{\delta}_{ik} + \theta_{jk} \tilde{\delta}_{il}, \quad (5)$$

where $\tilde{\delta}_{ij} = \delta_{ij} - \partial_i \partial_j / \nabla^2$. The induced term in the classical action is

$$\Delta L = 2\theta_{ki} \dot{h}_{ij} \left(\delta_{lj} - \frac{\partial_l \partial_j}{\nabla^2} \right) h_{kl} - 4h_{0j} \theta_{ln} \left(\delta_{jm} - \frac{\partial_j \partial_m}{\nabla^2} \right) \partial_l h_{nm}. \quad (6)$$

This term is invariant under gauge transformations satisfying $(\delta_{jm} - \partial_j \partial_m / \nabla^2) \xi_m = 0$. This (restricted) gauge freedom can be used to fix $\partial_m h_{mn} = 0$ and $h_{0m} = 0$, thus obtaining the simple modification $\Delta L = 2\theta_{ki} \dot{h}_{ij} h_{kj}$, which leads to the same modified dispersion relations we obtained before. Even if this deformation does not reproduce exactly the term ΔL , it leads to a modified gravity theory with (restricted) gauge invariance, and with the same deformed dispersion relations.

We notice that this is not the only Lorentz-violating additive term which can be obtained with use of the deformation of the Poisson algebra. As an example, if we assume $\tilde{\delta}_{ij} = \delta_{ij} \nabla^2 - \partial_i \partial_j$, we find that the following gauge-invariant term is generated in the action,

$$\begin{aligned} \Delta L = -\theta^p & \left[2\epsilon_{0kip} \dot{h}_{ij} (\delta_{lj} \nabla^2 - \partial_l \partial_j) h_{kl} \right. \\ & \left. - 4h_{0j} \epsilon_{0lnp} (\delta_{jm} \nabla^2 - \partial_j \partial_m) \partial_l h_{nm} \right], \end{aligned} \quad (7)$$

where $\theta_{ij} = -\epsilon_{0ijk} \theta^k$.

One may compare this with the well-known gravitational Chern-Simons term,⁷ $\Delta L^{CS} = 2\theta^\lambda h^{\mu\nu} \epsilon_{\alpha\mu\lambda\rho} \partial^\rho [\square h_\nu^\alpha - \partial_\nu \partial_\gamma h^\gamma{}^\alpha]$; if the θ^λ vector is chosen to be purely spacelike, and the axial gauge $h_{0m} = 0$ is imposed, the gravitational Chern-Simons term is reduced to

$$\Delta L^{CS} = -2\theta^l \dot{h}^{ij} \epsilon_{lki0} [\square h_j^k - \partial_j \partial_s h^{ks}], \quad (8)$$

which, under the gauge conditions $\partial_s h^{ks} = 0$ and $h_{0i} = 0$, is almost equivalent to Eq. (7), up to the term with third time derivative.

4. Summary

We have studied the consequences of a Lorentz-violating term in the propagation of gravitational waves in the linearized gravitation theory, finding a modified dispersion relation with a kind of birefringence effect. We have also shown how to generate this term via an appropriate deformation of the canonical algebra of the model, generalizing some ideas that have been proposed in connection with noncommutative gauge theories before.

Acknowledgments

This work is partially supported by the Brazilian agencies CNPq, FAPESP and CAPES. The work by A.Yu. P. is supported by the CNPq project 303461-2009/8.

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